

# Úlohy - komplexné čísla

1. Zapište v algebrickom tvare:

a)  $(3 + 2i)(2 - i);$  [8 + i]

b)  $(3 + 3i)(1 - i);$  [6]

c)  $\frac{2 + i}{1 + i};$  [3/2 - i/2]

d)  $\frac{1 + 2i}{2 - i} - \frac{2 - i}{1 + 2i};$  [2i]

2. Vyjadrite dané komplexné číslo v goniometrickom tvare:

a) 1; [1 · (cos 0 + i sin 0)]

b)  $\frac{1}{2} - \frac{\sqrt{3}}{2}i;$  [1 · (cos  $\frac{5\pi}{3}$  + i sin  $\frac{5\pi}{3}$ )]

c)  $-2\sqrt{2} - 2\sqrt{2}i;$  [4(cos  $\frac{5\pi}{4}$  + i sin  $\frac{5\pi}{4}$ )]

d)  $8\sqrt{3}i - 8;$  [16(cos  $\frac{2\pi}{3}$  + i sin  $\frac{2\pi}{3}$ )]

e)  $-3 + 3i;$  [3 $\sqrt{2}$ (cos  $\frac{3\pi}{4}$  + i sin  $\frac{3\pi}{4}$ )]

3. Určte reálnu a imaginárnu časť čísla z, ak

a)  $z = (1 + i)^5;$  [Re(z) = -4, Im(z) = -4]

b)  $z = (\sqrt{3} - i)^6;$  [Re(z) = -64, Im(z) = 0]

c)  $z = (1 + \sqrt{3}i)^4;$  [Re(z) = -8, Im(z) = -8 $\sqrt{3}$ ]

d)  $z = (-1 - \sqrt{3}i)^5;$  [Re(z) = -16, Im(z) = 16 $\sqrt{3}$ ]

e)  $z = (\sqrt{3} + i)^{33};$  [Re(z) = 0, Im(z) = -2<sup>33</sup>]

4. V goniometrickom tvare určte všetky hodnoty danej odmocniny:

a)  $\sqrt[3]{1 + \sqrt{3}i};$  [ $\sqrt[3]{2}$ (cos  $\frac{(1+2k)\pi}{9}$  + i sin  $\frac{(1+2k)\pi}{9}$ ),  $k \in \{0; 1; 2\}$ ]

b)  $\sqrt[5]{1};$  [(cos  $\frac{2k\pi}{5}$  + i sin  $\frac{2k\pi}{5}$ ),  $k \in \{0; 1; 2; 3; 4\}$ ]

c)  $\sqrt[4]{i};$  [(cos  $\frac{(1+4k)\pi}{8}$  + i sin  $\frac{(1+4k)\pi}{8}$ ),  $k \in \{0; 1; 2; 3\}$ ]

d)  $\sqrt[4]{-1 - i};$  [ $\sqrt[8]{2}$ (cos  $\frac{(5+8k)\pi}{16}$  + i sin  $\frac{(5+8k)\pi}{16}$ ),  $k \in \{0; 1; 2; 3\}$ ]

e)  $\sqrt[6]{-i};$  [(cos  $\frac{(3+4k)\pi}{12}$  + i sin  $\frac{(3+4k)\pi}{12}$ ),  $k \in \{0; 1; 2; 3; 4; 5\}$ ]